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13. ABSTRACT (Maximum 200 words)  Our work on simulation methodology begins with the formulation of a model as a stochastic process. We then use the structure of the stochastic process to develop methods for analyzing the output of the simulation. Diagram 1 is a block diagram which details our approach to system simulation. The performance evaluation and reliability analysis of complex engineering systems requires an ability to analyze mathematical models of these systems. Large-scale stochastic models are required to handle various uncertainties present in these systems. Unfortunately, the complexity of most stochastic models of real systems is well beyond our ability to apply classical mathematical analysis. Computer simulation of stochastic systems has become one of the principal alternatives to classical analysis. The main thrust of our research is improving the efficiency of simulation methods and extending their applicability to wider class of stochastic systems. We also carry out research on a variety of stochastic models with the aim of obtaining closed form solutions or approximations.					
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# **SIMULATION METHODOLOGY**

by

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and

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**FINAL REPORT**

August 1998

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of

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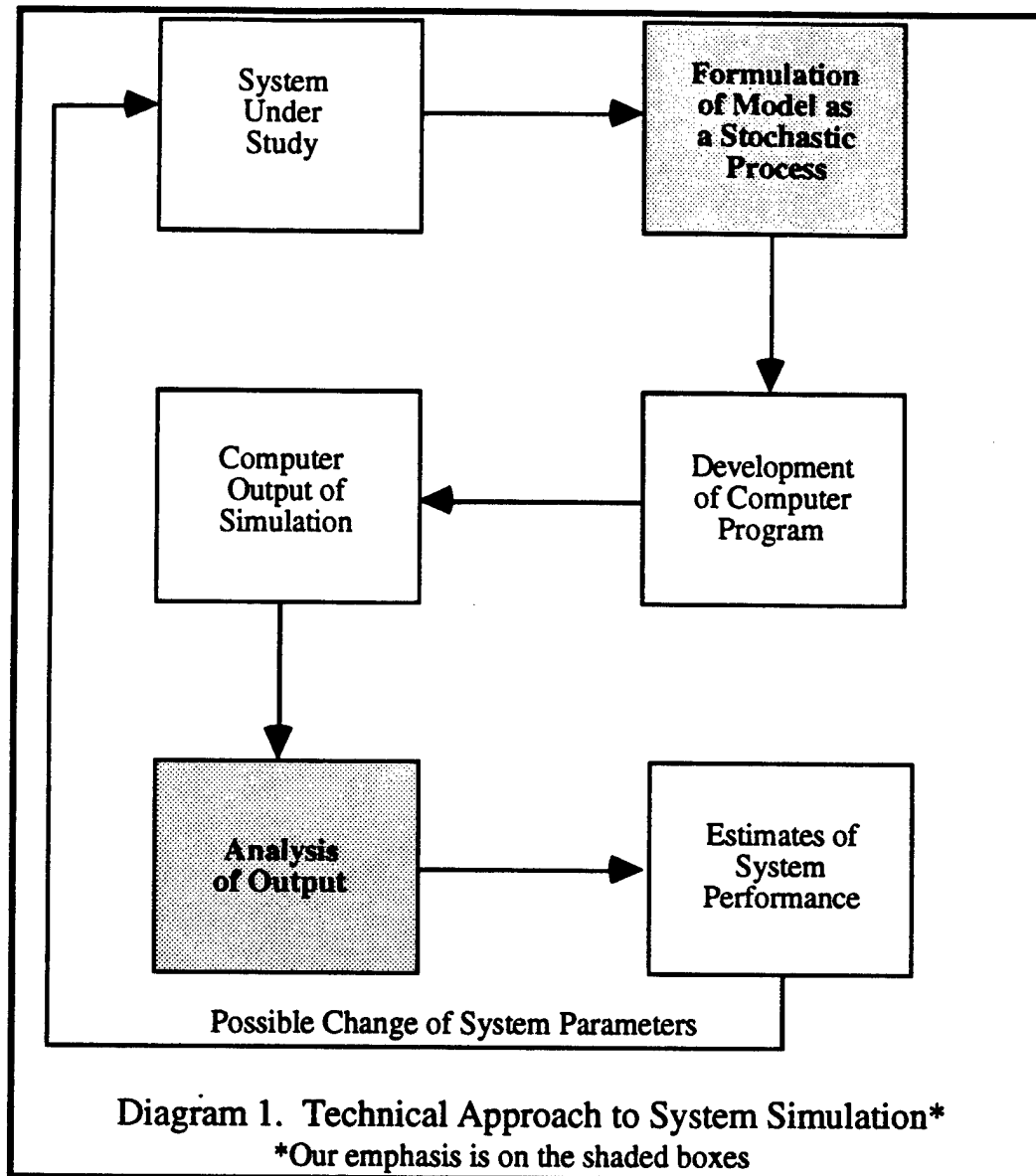
## **1. INTRODUCTION**

This contract covered the period April 15, 1994 through December 31, 1997. It supported Professor Peter W. Glynn for two months in the summer and Professor Donald L. Iglehart for one month in the summer.

Our work on simulation methodology begins with the formulation of a model as a stochastic process. We then use the structure of the stochastic process to develop methods for analyzing the output of the simulation. Diagram 1 is a block diagram which details our approach to system simulation. The performance evaluation and reliability analysis of complex engineering systems requires an ability to analyze mathematical models of these systems. Large-scale stochastic models are required to handle various uncertainties present in these systems. Unfortunately, the complexity of most stochastic models of real systems is well beyond our ability to apply classical mathematical analysis. Computer simulation of stochastic systems has become one of the principal alternatives to classical analysis. The main thrust of our research is improving the efficiency of simulation methods and extending their applicability to wider class of stochastic systems. We also carry out research on a variety of stochastic models with the aim of obtaining closed form solutions or approximations.

## **2. IMPORTANT RESULTS OBTAINED**

High speed communications networks of the future are expected to use the asynchronous transfer mode (ATM) protocol to simultaneously carry data, voice, and video traffic. This traffic flows into a switch which is modeled as a single serve in a queueing network. The design and control of such networks involves the appropriate sizing of buffers used to store message packets. The asymptotic decay rate for the tail of the steady-state waiting time distribution has been derived. This decay rate may be used to create effective bandwidths for admission control and other network resource allocation problems. These rates can also be used to speed up simulations of the network. Steady-state performance measures for highly dependable systems (such as fault-tolerant computer systems) are estimated by simulations involving rare events. Earlier work dealt with Markovian systems that could be formulated as regenerative processes. Our current work assumes that repair and failure times are non-exponential and this leads to non-Markovian systems for which the regenerative structure is lost. A new approach has been developed using importance sampling applied to a sequence of cycles which are no longer independent, identically distributed as in the regenerative case. Experimental results show that the method is effective in practice. Techniques have been developed for estimating the gradient of performance measures with respect to design parameters. These gradient estimates are required for sensitivity analyses and system optimization. System simulations are a natural candidate for parallel processing. A simple way to exploit the power of parallel processing is to run multiple independent replications in parallel and average the results. Whether estimating transient or steady-state quantities, a number of problems arise that can lead to incorrect estimates. We have developed a number of techniques for overcoming these problems. Solutions of Poisson's equation provide the key to studying the bias and variability of additive functionals of a Markov



chain. These solutions have been obtained for discrete and continuous time Markov chains, and for the waiting time sequence of the single server queue.

The maximum of a random walk over the non-negative integers is an important random variable that arises in insurance risk theory, sequential analysis, and queueing theory. We developed a method for simulating the exact distribution of this maximum along with several ways to reduce the variance of our estimator. In manufacturing environments one is often concerned with optimizing the performance of a system over a set of decision parameters, when the performance can only be evaluated through simulation. We study how fast the number of simulation replications must grow as the number of grid points in the decision space increases. For non-irreducible, general state space Markov chains we developed a criteria for guaranteeing tightness of the chains. Tightness is a condition that guarantees that Markov chains are well behaved in the limit as time becomes large. The shortest processing times scheduling rule is shown to be asymptotically optimal when processing times are exchangeable across machines and independent across jobs. In steady-state simulations estimating the time-average variance constant (TAVC) is critical for obtaining confidence intervals of performance measures. Several methods for estimating the TAVC have been developed in the literature. We have studied the computational complexity of these estimates for the various methods. A continuous time Markov process is governing by its infinitesimal generator. An important problem when observing financial time series is to estimate this generator when the process is only observable at random times. We developed a family of generalized method-of-moment estimators for this problem, the maximum-likelihood estimator being a special case. We established strong consistency, asymptotic normality, and characterization of the standard errors for these estimators.

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### 3. TECHNICAL REPORTS

94-6	Efficiency Improvement Techniques, April 1994	Peter W. Glynn
94-7	A Family of Regenerative Moment Identities, May 1994	James M. Calvin Peter W. Glynn
94-10	Increasing the Frequency of Regeneration for Markov Processes, August 1994	Sigrun Andradottir James M. Calvin Peter W. Glynn
94-11	Large Deviations for the Infinite Server Queue in Heavy Traffic, August 1994	Peter W. Glynn
94-12	Accelerated Regeneration for Markov Chain Simulations, September 1994	Sigrun Andradottir James M. Calvin Peter W. Glynn
94-13	A Martingale Approach to Regenerative Simulation, September 1994	Peter W. Glynn Donald L. Iglehart
94-18	Average Case Behavior of Random Search for the Maximum, December 1994	James M. Calvin Peter W. Glynn
94-19	Complexity of Non-Adaptive Optimization Algorithms for a Class of Diffusions, December 1994	James M. Calvin Peter W. Glynn
95-1	Two Approaches to the Initial Transient Problem, April 1995	Peter W. Glynn
95-2	Performance Analysis of Future Event Sets, May 1995	Halim Damerdj Peter W. Glynn
95-4	Efficient Simulation via Coupling, June 1995	Peter W. Glynn Eugene W. Wong
95-5	Some New Results on the Initial Transient Problem, June 1995	Peter W. Glynn
95-6	A Set of Extensions to the SIMAN/ARENA Simulation Environment, August 1995	Marcelo J. Torres Peter W. Glynn
96-6	Limit Theory for Performance Modeling of Future Event Set Algorithms, June 1996	Halim Damerdj Peter W. Glynn
96-7	Derandomizing Variance Estimators, June 1996	Shane Henderson

		Peter W. Glynn
96-9	Nonparametric Estimation of Tail Probabilities for the Singer Server Queue, July 1996	Peter W. Glynn Marcelo Torres
96-10	Grid-Based Simulation and the Method of Conditional Least Squares	Peter W. Glynn Kathy Ensor
96-14	Numerical Computation of Large Deviations Exponents Via Simulation	Peter W. Glynn
97-1	Simulating the Maximum of a Random Walk February 1997	Peter W. Glynn Katherine Ensor
97-2	Stochastic Optimization via Grid Search, February 1997	Peter W. Glynn Katherine Ensor
97-4	Tightness for Non-irreducible Markov Chains, June 1997	Peter W. Glynn Sean Meyn
97-5	On the Asymptotic Optimality of the SPT Rule for the Static Flow Shop Average Completion Time Problem, June 1997	Cathy J. Xia G. Shanthikumar Peter W. Glynn
97-7	Computational Efficiency Evaluation in Output Analysis, July 1997	Halim Damerdj S. Henderson Peter W. Glynn
97-10	Estimation of Continuous-Time Markov Processes Sampled at Random Time Intervals, July 1997	Darrell Duffie Peter W. Glynn

## 5. SCIENTIFIC PERSONNEL

The following scientific personnel were supported on this contract.

### Faculty supported:

Peter W. Glynn, Professor and co-Principal Investigator

Donald L. Iglehart, Professor co-Principal Investigator

### Honors

Peter W. Glynn was appointed the Thomas Ford Professor of Engineering in 1996.